Research Awards Screening Test, 2006

KEY

Section 1: Algebra

1.1 a, b, c

1.2 b, c

1.3 c

1.4 0,5,8,9

1.5 c

1.6 12

1.7 Any matrix of the form: (a, b and c all non-zero)

$$\begin{bmatrix} a & b & c/2 \\ 0 & b & c \\ 0 & 0 & c/3 \end{bmatrix}$$

1.8 a, b

1.9 b, c

1.10 b, c, d

Section 2: Analysis

2.1 $C = \cap_{n=1}^{\infty} A_n$

2.2 $2n \int_0^x (2x-t)^{n-1} f(t)\ dt + x^n f(x)$

2.3 a

2.4 b

2.5 b, c

2.6 $b\frac{\partial g}{\partial x} = a\frac{\partial g}{\partial y}.$

2.7 $\left(\frac{1}{2^{\frac{1}{3}}}, -\frac{1}{2^{\frac{1}{3}}}\right).$

2.8 $e^{-\frac{1}{k}}.$

2.9 All integers

2.10 $\left[\left(\frac{\partial f}{\partial x}(x_0, y_0, z_0)\right)^2 + \left(\frac{\partial f}{\partial y}(x_0, y_0, z_0)\right)^2 + \left(\frac{\partial f}{\partial z}(x_0, y_0, z_0)\right)^2\right]^{\frac{1}{2}}$

Section 3: Topology

3.1 (i) continuous at all irrationals, (ii) continuous only at $t = 1$

3.2 $f(a) = g(a)$

3.3 A and B have the same cardinality

3.4 (i) $f(D)$ is necessarily an interval; (ii) $[a, b]$

3.5 (i) X_α is connected if and only if $\alpha \leq \frac{3}{4}$. (ii) When not connected, it has 3 components

3.6 X_2 and X_4 are homeomorphic

3.7 Compact sets are X_2 and X_3

3.8 Locally compact sets are X_1, X_2 and X_3

3.9 Complete metric spaces are X_1, X_2 and X_3

3.10 X_2

Section 4: Applied Mathematics

4.1 $\frac{3}{4}\sqrt{\pi}$

4.2 0

4.3 $\frac{12\pi}{5}a^6$

4.4 3

4.5 b, c

4.6 $c = 2$

4.7 elliptic in the region $\{(x, y) \in \mathbb{R}^2 \ : \ y < 0\}$

4.8 $\pi/5$

4.9 a, b, d

4.10 A linear functional in 3 variables with coefficients 6, -6 and -4; example: $6w_1 - 6w_2 - 4w_3$

Section 5: Miscellaneous

5.1 m and n are coprime

5.2 0

5.3 $n(n-1)2^{n-2}$

5.4 2^{-7}

5.5 2

5.6 there is no solution

5.7 $[0, 1/2]$ for each k

5.8 Yes; if $K = \{a_1, \ldots, a_n\}$, then take $(x - a_1) \ldots (x - a_n) + 1$, for example.

5.9 $2e$

5.10 a, c