Homework 2

MA 216: Graph Theory
Autumn 2019
Indian Institute of Science

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Submit only the starred (*) problems by Sep. 5. Unless otherwise stated n is the number of vertices and m is the number of edges of the graph in the question.

1. Recall that δ is the minimum degree of vertices in a graph. Show that every simple graph G contains a path of length δ.
2. (*) Show that every simple graph G with $\delta \geq 2$ contains a cycle of length at least $\delta + 1$.
3. (*) Show that any two longest paths in a connected graph have a vertex in common.
4. Give an algorithm to decompose K_{2n+1} into Hamiltonian cycles.
5. (*) Recall that for $X \subset V(G)$, $d(X) = \#\partial(X)$. For a loopless graph G, prove that if $X, Y \subset V(G)$, $d(X) + d(Y) \geq d(X \cup Y) + d(X \cap Y)$.
6. Prove that is there is an $xy-$walk in a graph, there is also an xy-path.
7. (*) Let A be the adjacency matrix of G. Show that the number of $xy-$walks of length k in G is the (x, y)'th entry of A^k.
8. Prove that a connected even graph does not have any bridges.
9. Suppose n is even and m is odd. Either give an example of such an Eulerian graph or prove that no such Eulerian graph exists.
10. Suppose G is simple with $n \geq 3$ in which the degree sum of any two nonadjacent vertices is at least n. Prove that G is Hamiltonian.
11. (*) Suppose G is simple and $\delta \geq (n-1)/2$. Prove that G is traceable.
12. (*) Recall that Δ is the maximum degree of vertices in a graph. Show that every tree T has at least Δ_T leaves.