

Homework 3

MA 216: Graph Theory
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Indian Institute of Science

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Submit only the starred (*) problems by Sep. 19. Unless otherwise stated n is the number of vertices and m is the number of edges of the graph in the question.

1. Prove that that number of spanning trees of a graph G is given by $|\det L_G^{i,j}|$, where $L_G^{i,j}$ is the matrix obtained from the Laplacian, L_G by deleting the i 'th row and the j 'th column.
2. (*) Show that a sequence of integers (d_1, \dots, d_n) is the sequences of degrees of the vertices of a tree iff $\sum_{i=1}^n d_i = 2(n-1)$.
3. Prove that in any tree T , all longest paths cross each other in one vertex.
4. (*) Let G be a graph and $e \in E(G)$. Then prove that the number of spanning trees of G is the sum of those of $G \setminus e$ and $G \cdot e$ (contraction).
5. (*) Let A_n be the graph obtained from K_n by deleting an edge. Find the number of spanning trees of A_n .
6. (*) Show that every nontrivial graph has at least two non-cut vertices.
7. Let G be a connected graph on at least three vertices and $e = (uv)$ be a bridge. Show that either u or v is a cut vertex of G .
8. (*) Prove that G is even iff each of its blocks is even.
9. Prove that the symmetric difference of two even subgraphs of a graph is an even subgraph.
10. (*) Show that if G has no even cycles, every block of G is either an odd cycle or a copy of K_1 or K_2 .
11. Let F be a nonseparable proper subgraph of a graph G and P be an ear of F . Then show that $F \cup P$ is nonseparable.