Homework 4

MA 216: Graph Theory
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Indian Institute of Science

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Submit only the starred (*) problems by Oct. 31. Unless otherwise stated n is the number of vertices and m is the number of edges of the graph in the question.

1. Let f and f' be two flows of equal value in a network N. Show that $f - f'$ is a circulation in N.
2. (*) Recall that Q_n is the graph of the n-cube. Find a maximum collection of edge-disjoint and pairwise internally disjoint paths in Q_n from $x = (0, \ldots, 0)$ to $y = (1, \ldots, 1)$. Then find the minimum vertex cut separating x and y.
3. If G is simple with $\delta \geq n - 2$, then prove that $\kappa(G) = \delta$.
4. (*) If G is simple with $n \geq k+1$ and $\delta \geq (n+k-2)/2$, then prove that G is k-connected.
5. Prove that $K_{3,3}$ is nonplanar constructively (as was done in class for K_5).
6. (*) Prove that every graph is embeddable in $\mathbb{R}^3$.
7. (*) In any connected plane graph G, prove that the boundary of a face is a closed walk in which every bridge of G is traversed twice.
8. Recall that G^* is the planar dual of G. Prove that $G^{**} \cong G$ iff G is connected.
9. (*) If F is a graph with $\Delta \leq 3$, then prove that any graph which has an F-minor also has an F-subdivision.
10. If G and G' are planar graphs which intersect at K_2, prove that $G \cup G'$ is planar.
11. (*) Without using either Wagner's or Kuratowski's theorems, show that
    (a) If G contains a $K_{3,3}$ minor, it has a $K_{3,3}$ subdivision, and
    (b) If G contains a K_5 minor, it has either a K_5 subdivision or a $K_{3,3}$ subdivision.