Homework 5

MA 216: Graph Theory
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Instructor: Arvind Ayyer

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1. Find the number of perfect matchings of the Petersen graph.
2. Find the number of perfect matchings of K_{2n} and $K_{n,n}$.
3. Suppose G is a graph with all vertices having odd degree. Show that every perfect matching M includes every bridge of G.
4. Find a graph G such that $\#N(S) \geq \#S$ for all $S \subseteq V(G)$ and yet G has no perfect matching.
5. Let S and T be maximum independent sets in a graph G. Show that the induced subgraph $G[S \Delta T]$ has a perfect matching.
6. Let G be a graph and $S \subsetneq V(G)$. Then show that $o(G - S) - \#S \equiv n \pmod 2$.
7. Show that the empty set is a barrier of every graph without essential vertices.
8. Show that a tree T has a perfect matching iff $o(T - v) = 1$ for all $v \in V(T)$.
9. For the Fibonacci polynomials $F_n(x)$, prove the following:
   (a) $\displaystyle\sum_{n=0}^{\infty} F_n(x) t^n = \frac{t}{1 - xt - t^2}$
   (b) $\displaystyle F_n(x) = \frac{z_+^n - z_-^n}{z_+ - z_-}$, where $\displaystyle z_\pm = \frac{x \pm \sqrt{x^2+4}}{2}$.
   (c) $\displaystyle F_n(x) = \sum_{k=0}^{n} \binom{(n+k-1)/2}{k} x^k$
10. Find a bipartite graph on $2n$ vertices and an ordering of these vertices such that the greedy algorithm uses n colours (instead of 2).
11. Find a graph G whose chromatic polynomial is $P(G, k) = k(k-1)^{n-1}$.