# Homework 2

MA 368A: Exclusion processes
Autumn 2024
Indian Institute of Science

Instructor: Arvind Ayyer

October 15, 2024

Submit only the starred (*) problems by Nov. 3.

1. (*) Let $F = xD + zE$, where $x, z$ are indeterminates. Show that
$$\langle W|_1 F^n |V\rangle_1 \sim \frac{(\sqrt{x}+\sqrt{z})^{2n+3}}{2\sqrt{\pi} n^{3/2} (x\,z)^{3/4}}.$$

2. (*) Prove that the partition function for the 2-ASEP on the ring is a palindromic polynomial of $q$, namely that the coefficients read the same forwards and backwards.

3. (*) What is the problem with this argument? The current of 1's in the 2-ASEP on the ring is always 0 because it is proportional to
$$\mathrm{Tr}((AE + qAD - qEA - DA)\dots),$$
which is 0 by the 2-ASEP algebra.

4. Prove that if $n_0 = n_2$, the current of 1's is 0 in the 2-ASEP.

5. Prove the particle-hole symmetry for the semipermeable TASEP.

6. (*) Prove the following identity for the ballot numbers,
$$\sum_{k=b}^{a} C_{a-k} C_{k-b}^{k+d} = C_{a-b}^{a+d+1}.$$

7. Use the formula for $\langle \xi_i \rangle$ proved in class to obtain a formula for $\langle \eta_i \rangle$ in the semipermeable TASEP.