MATH 224 : COMPLEX ANALYSIS
SPRING 2026
HOMEWORK 6

Instructor: GAUTAM BHARALI **Assigned: FEBRUARY 12, 2026**

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1. Let $\Omega \subsetneq \mathbb{C}$ be a star-shaped domain that is **not** convex and let $a \in \Omega$ be such that $[a, z] \subsetneq \Omega$ for each $z \in \Omega$. Let $\gamma : [0, 1] \longrightarrow \Omega$ be a piecewise-$\mathcal{C}^1$ path in Ω such that $a \notin \langle\gamma\rangle$. Show that γ is homotopic to $\mathsf{const}_{\gamma(0)}$.

2. Let Ω be a non-empty open subset of $\mathbb{C}$ and let $a, b \in \Omega$. Let $\gamma_j : ([0, 1], 0, 1) \longrightarrow \Omega$, $j = 0, 1$, be two paths in Ω such that $\gamma_0 \sim_{\text{FEP}} \gamma_1$. Let $\Gamma : [0, 1] \times [0, 1] \longrightarrow \Omega$ be a homotopy between γ_0 and γ_1. Elaborate upon the strategy of considering the paths

$$\big(\Gamma(\cdot, 0)|_{[0,1-t]}\big) * \big(\Gamma(1-t, \cdot)|_{[0,1-t]}\big) * \big(-\big(\Gamma(\cdot, 1)|_{[0,1-t]}\big)\big), \quad t \in [0, 1]$$

— where the second path is traversed in t units of "time" — discussed in class to establish that $\gamma_0 * (-\gamma_1) \sim \mathsf{const}_a$.

3–5. Problems 5–7 from the exercises to IV–Section 6 of Conway.