# MATH 224 : COMPLEX ANALYSIS
## SPRING 2026
## HOMEWORK 9

**Instructor: GAUTAM BHARALI** **Assigned: MARCH 13, 2026**

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**1.** (Carried over from Homework 8) Problem 13 from the exercises to V–Section 1 of Conway.
**Note.** This problem is **important.** Firstly, it introduces certain new definitions. Secondly, it forms the basis for Problem 9 below.

**2.** Let $f(z) = 1/(z-1)(z-2)$.

(*a*) Using **purely elementary** methods (i.e., do **not** appeal to the integral formula for the Laurent coefficients), compute the Laurent expansions of $f$ around $z = 1$ and $z = 2$.

(*b*) Give the regions in which these two expansions are absolutely convergent.

**3.** Suppose $f$ is holomorphic in the punctured disc $D(a;\delta)^*$ and suppose $a$ is a pole of $f$ of order $m \geq 1$. Then, show that

$$\operatorname{Res}(f;a) \ = \ \frac{1}{(m-1)!} g^{(m-1)}(a),$$

where $g(z) := (z-a)^m f(z)\ \forall z \in D(a;\delta)^*$.

**4. Carefully review Examples 2.5, 2.7, 2.9, 2.10 and 2.12** in Chapter V of Conway's book to understand some important classes of tricks (of which the tricks represented by Example 2.5 and Example 2.9 were discussed in class) for applying the Residue Theorem to evaluate **real** improper integrals.

**5.** Consider the improper integrals

$$I_n \ := \int_0^\infty \frac{dx}{1+x^n}, \quad n = 2, 3, 4, \dots .$$

In view of the discussion in class, you may **assume without proof** that these improper integrals exist.

(*a*) Compute $I_n$ for $n = 2, 4, 6, \dots$ using the ideas presented in class or in Example 2.5 in Chapter V of Conway.

(*b*) For odd $n$, one can choose a **different** closed piecewise-$\mathcal{C}^1$ path that enables one to compute the relevant $I_n$'s and involves computing a lot fewer residues. Determine what this path is and compute all $I_n$ for odd $n$.

**6.** Choosing appropriate closed piecewise-$\mathcal{C}^1$ paths in $\mathbb{C}$, work out the **real** integrals given in problems 2(*c*), 2(*d*), 2(*e*) and 2(*g*) from the exercises to V–Section 2 of Conway.

**7–9.** Problems 6, 7, and 12 from the exercises to V–Section 2 of Conway.

**10.** Read the passage on the second page of V–Section 2 of Conway's book to know why the Argument Principle has this name. (**Caution:** Providing all the details underlying the latter discussion — which can be done — leads to a proof that is a lot more cumbersome than the proof that is generally considered standard nowadays.)