UM 101 : ANALYSIS & LINEAR ALGEBRA – I

“AUTUMN” 2020

HOMEWORK 3

Instructor: GAUTAM BHARALI **Assigned: DECEMBER 3, 2020**

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1. Let $\mathbb{F}$ be an ordered field and let $S \subseteq \mathbb{F}$. If S has a least upper bound, then show that it is unique.

2. (Apostol, I-3.12, Prob. 2) Let x be an arbitrary real number. Show that there exist integers m and n such that $m < x < n$.
Clarification. The set of integers is the set $\mathbb{N} \cup \{-n : n \in \mathbb{P}\}$, where $-n$ is the negative of n viewed as an element of $\mathbb{R}$.
Hint. It can useful to consider Theorem I.28 in Apostol.

3. Let $\{a_n\} \subset \mathbb{R}$ and let $L \in \mathbb{R}$. How do you express quantitatively the statement, “$\{a_n\}$ does **not** converge to L”?

4. Let $\{a_n\}$ be a convergent sequence with limit L. Prove that the sequence $\{b_n\}$, where

$$b_n = \frac{a_1 + \cdots + a_n}{n},$$

converges to L.

The following problem will go a little beyond what has been taught until now. You will need the results from the beginning of the **lecture of December 4** to solve it.

5. For each of the following sequences, determine whether it converges or diverges. **Justify** your answer.

a) $\left\{\dfrac{10^7 n}{4n^2 - 4n + 1}\right\}$

b) $\{1 + (-1)^n\}$

c) $\{\sqrt{n+1} - \sqrt{n}\}$

d) $\{(1 + (-1)^n)/n\}$

e) $\left\{\dfrac{n^2}{n+5}\right\}$

f) $\left\{\dfrac{\sqrt{n}\cos(n!)\sin(1/n!)}{n+1}\right\}$

Tip. In those cases where you think the sequence is divergent, it is useful to **assume** that it has the limit L — where L is an arbitrary real number — and arrive at a contradiction.