UM 101 : ANALYSIS & LINEAR ALGEBRA – I

"AUTUMN" 2020

HOMEWORK 5

Instructor: GAUTAM BHARALI **Assigned: DECEMBER 17, 2020**

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1. State whether or not each of the following non-negative series converges. Give **justifications.**

a) (Apostol, 10.16, Prob. 13) $\sum_{n=1}^{\infty} \frac{n^3\left(\sqrt{2}+(-1)^n\right)^n}{3^n}$

b) $\sum_{n=1}^{\infty}(n!)^2/(2n)!$

Note. You must use **only** the tests and results discussed in class or assigned for self-study.

2. Let p be a real number contained in an open interval I. Let f be a $\mathbb{R}$-valued function such that $f(x)$ is defined at each $x \in I$ except perhaps at $x = p$. Let $A \in \mathbb{R}$. How do you express quantitatively (involving parameters like ε, etc., in an appropriate way) the statement, "$f(x)$ does **not** have the limit A as x approaches p"?

3. Show that

$$\lim_{x \to 0} \frac{\sin(6x) - \sin(5x)}{x}$$

exists. Give **justifications** in terms of the limit theorems that are used.

Note. You may use standard trigonometric identities learnt in high school **without** deriving them.

4. Let n be some (fixed) positive integer and let $p \in \mathbb{R}$. Complete the following outline to show that $\lim_{x \to p} x^n = p^n$ using **only** the "ε-δ definition".

a) Establish the desired limit for the case $n = 1$ using the "ε-δ definition".

b) Now, use Part (*a*) to establish the stated limit.