# UM 101 : ANALYSIS & LINEAR ALGEBRA – I

**"AUTUMN" 2020**

**HOMEWORK 9**

**Instructor: GAUTAM BHARALI** **Assigned: JANUARY 28, 2021**

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**1.** Let $a < b$ be real numbers and let $s : [a, b] \to \mathbb{R}$ be a step function.

*a*) Prove that $s$ is integrable according to the abstract definition given in terms of upper and the lower integrals.

*b*) You have been given a **formula** for the integral of a step function on $[a, b]$. Show that the value of the integral of $s$ given by the above-mentioned definition agrees with that given by the formula.

**2.** Let $f$ be a function defined on an interval $[-A, A]$, $A > 0$, and suppose $f|_{[0,A]}$ is Riemann integrable. Suppose $f$ is an even function (i.e., $f(x) = f(-x)$ for any $x \in [-A, A]$). Prove that $f$ is integrable and show that

$$\int_{-A}^{A} f(x)dx = 2\int_{0}^{A} f(x)dx.$$

**3.** Let $a < b$ be real numbers and let $f : [a, b] \to \mathbb{R}$ be Riemann integrable on $[a, b]$. Show that for any $c, d \in \mathbb{R}$ such that $a \leq c < d \leq b$, $f|_{[c,d]}$ is Riemann integrable on $[c, d]$.

**4.** Let $a < b$ be real numbers. Use the fact that if a function $f : [a, b] \to \mathbb{R}$ is continuous, then it is **uniformly** continuous, to prove the Small Span Theorem.

**5.** Let $n \in \mathbb{N} - \{0, 1\}$. Define $f_n : \mathbb{R} \to \mathbb{R}$ as $f_n(x) = x^n$ for each $x \in \mathbb{R}$. Show that $f_n$ is **not** uniformly continuous.