The third Indo-Russian meeting in probability and statistics
8–12 January, 2018
Venue: LH-1, Department of mathematics, Indian Institute of Science

Each day: 8th, 9th, 11th, 12th (no talks on 10th).

9:30-10:45	10:45-11:10	11:10-12:55	12:50-2:15	2:15-3:30	3:30-3:55	4:10-5:55
Course 1	Coffee	3 talks	Lunch	Course 2	Coffee	3 talks

Course 1: Dmitry Zaporozhets on *Stochastic geometry*

Course 2: Parthanil Roy on *Stable random fields*

Short talks: Each short talk is of 35 minutes duration (including question time)

Day	Morning (11:10-12:55)	Afternoon (4:10-5:55)
	Fedor Petrov	Moumanti Podder
8th	Riddhipratim Basu	Kseniya Volkova
(Monday)	– No talk –	Anish Sarkar
	Apoorva Khare	Kavita Ramanan
9th	Iuliia Petrova	Himanshu Tyagi
(Tuesday)	Anna Gusakova	Denis Dimitrov
10th	**BREAK**	**No lectures**
	Anup Biswas	Vladislav Vysotsky
11th	Sergey Berezin	Fedor Sandomirskiy
(Thursday)	Sourav Sarkar	Mathew Joseph
	Mariia Platonova	Alexander Bufetov
12th	Deepan Basu	Probal Chaudhuri
(Friday)	Rishideep Roy	– No talk –

Titles and abstracts of courses

Course 1: Dmitry Zaporozhets on *Stochastic geometry*

We will consider several classic models of stochastic geometry. After that we will talk about some not widely known connections between probability theory and convex geometry. A number of open problems will be formulated and discussed.

Course 2: Parthanil Roy on *Stable random fields*

In this mini course, the strong interplay between ergodic theory (of quasi-invariant group actions), algebra (of finitely generated abelian groups) and probability theory (of stationary symmetric stable random fields) will be discussed. If time permits, a newly discovered connection to von Neumann algebra (of group measure space construction) will also be presented along with several open problems.

The material for the first part of this mini course can be found in https://arxiv.org/abs/1702.00393. Click here for more details.

Titles and abstracts of short talks

Fedor Petrov (St. Petersburg State University and V.A. Steklov Institute of Mathematics)
Probabilistic aspects of polynomial combinatorics
Several recent breakthrough results in additive combinatorics, first of all Croot, Lev, Pach theorem and method, relies both on polynomial and probabilistic arguments. I want to survey this intensively developing area for a probabilistic audience.

Riddhipratim Basu (ICTS, Bangalore)
Concentration of Zeroes of Stationary Gaussian Processes
For a centred stationary Gaussian process on the real line whose spectral measure has finite second moment, the mean number of zeroes is given by the well-known Kac-Rice formula. We show that under certain additional regularity assumptions, the number of zeroes in an interval is concentrated around its mean with failure probability that is exponentially small in the length of the interval. Joint work with Amir Dembo, Naomi Feldheim and Ofer Zeitouni.

Moumanti Podder (Georgia tech, USA)
Sofic and percolative entropies of Gibbs measures on regular infinite trees
Consider a statistical physical model on the d-regular infinite tree T_d described by a set of interactions Φ. Let $\{G_n\}$ be a sequence of finite graphs with vertex sets V_n that locally converge to T_d. Let μ_n be the Gibbs measure on G_n constructed via pull-back from Φ. Assume $\{\mu_n\}$ converges to a Gibbs measure μ on T_d in the local weak* sense, and let Φ exhibit strong spatial mixing. We show that the limit of the specific entropies $|V_n|^{-1}H(\mu_n)$ is equal to the *percolative entropy*

$H_{perc}(\mu)$. We emphasize here that the percolative entropy is a different quantity from the much more commonly studied Bethe-Ansatz limit.

Kseniya Volkova (Saint-Petersburg State University)
Goodness-of-fit tests for the uniform law based on the characterization by the ratio of order statistics, and their efficiencies.
We construct integral and supremum type goodness-of-fit tests for the uniform law based on Ahsanullah's characterization of the uniform law. We discuss limiting distributions of new tests and describe the logarithmic large deviation asymptotics of test statistics under the null-hypothesis. This enables us to calculate their local Bahadur efficiency under some parametric alternatives. Conditions of the local optimality of new statistics are given. Joint work with M. S. Karakulov and Ya. Yu. Nikitin.

Anish Sarkar (ISI, Delhi)
The $2d$-directed spanning forest converges to the Brownian web
The two-dimensional directed spanning forest (DSF) introduced by Baccelli and Bordenave is a planar directed tree whose set of vertices is a homogeneous Poisson point process $\mathcal{N}$ on $\mathbb{R}^2$. If the DSF has direction $-e_y$, the ancestor $h(\mathbf{u})$ of a vertex $\mathbf{u} \in \mathcal{N}$ is the nearest Poisson point (for the L_2 distance) having strictly larger y-coordinate. We show that when properly scaled, the family of its paths converges in distribution to the Brownian web. This verifies a conjecture made by Baccelli and Bordenave in 2007. A key ingredient for the proof is to control the tail distribution of the coalescing time between two paths of the DSF. The facts that the DSF spans on a Poisson point process on the plane and that its construction is based on the L^2 distance - which is very natural - destroys all Markov and martingale properties on which the existing literature usually relies for proving the convergence of discrete forests to the Brownian web.

Apoorva Khare (IISc, Bangalore)
Probability inequalities over metric semigroups
We explore two classical inequalities and how they can be extended beyond the Banach space setting. The former, the Hoffmann-Jørgensen inequality, can be stated using only the notions of a binary associative operation and a distance function. Thus, we prove a generalized variant that (i) holds in a very primitive mathematical setting: metric semigroups. Additionally, we show how our result (ii) extends state-of-the-art even for the simplest case of real numbers, and (iii) simultaneously unifies several variants in the literature, which were not previously reconciled. We then study the Khinchin–Kahane inequality, and prove two variants: one for all abelian metric groups; and a second, sharp version for all such groups which are also "normed".

Iuliia Petrova (Saint-Petersburg State University)
Exact small ball asymptotics in L_2-norm for finite dimentional perturbations of Gaussian processes

We consider the problem of small ball behavior in L_2-norm for some finite dimentional perturbations of Gaussian processes. In noncritical case the explicit relation between exact small ball asymptotics for initial and perturbed processes is obtained. In critical case, if the perturbation is $\langle\langle$good$\rangle\rangle$ enough, we can obtain a similar result. Also we consider some natural examples from statistics (processes of Durbin).

Anna Gusakova (Institute of Mathematics of National Academy of Science of Belarus and Bielefeld University)
Distribution of complex algebraic numbers on the unit circle
The question of the distribution of real and complex algebraic numbers has been considered during the last few years and a tight relation between the distribution of algebraic numbers and the distribution of the zeros of random polynomials has been computed. The typical problem is to find the asymptotic formula for the number of algebraic numbers lying in some domain D with given degree n and height bounded by Q when Q tends to infinity. In this talk we introduce such formula for D being the arc of the unit circle in the complex plane. We will use the connection between the distribution of algebraic numbers and the distribution of the zeros of random trigonometric polynomials to prove our result.

Kavita Ramanan (Brown university, USA)
Tales of Random Projections
The interplay between geometry and probability in high-dimensional spaces is a subject of active research. Classical theorems in probability theory such as the central limit theorem and Cramer's theorem can be viewed as providing information about certain scalar projections of high-dimensional product measures. In this talk we will describe the behavior of random projections of more general (possibly non-product) high-dimensional measures, which are of interest in diverse fields, ranging from asymptotic convex geometry to high-dimensional statistics. Although the study of (typical) projections of high-dimensional measures dates back to Borel, only recently has a theory begun to emerge, which in particular identifies the role of certain geometric assumptions that lead to better behaved projections. We will review past work on this topic, including a striking central limit theorem for convex sets, and show how it leads naturally to questions on the tail behavior of random projections and large deviations on the Stiefel manifold. The talk will be based on various joint works with Steven Kim, some also involving Nina Gantert.

Himanshu Tyagi (IISc, Bangalore)
A Change of Measure Approach for Proving Strong Converse Theorems in Information Theory
A strong converse theorem in information theory shows that allowing a nonvanishing error in a compression or transmission problem does not lead to a gain in asymptotic rate. Recently, we proposed a new approach for proving strong using the continuity of Shannon entropy in Wasserstein distance. Using this approach, we can recover most known strong converse results, often with

much shorter proofs. Furthermore, we can prove a strong converse for the communication complexity of function computation, which was open prior to this work. In this talk, we will review this basic approach focusing on a simple example and will give pointers for application in more complicated models.

This is joint work with Shun Watanabe (Tokyo University of Agriculture and Technology).

Denis Dimitrov (Lomonosov Moscow State University)
Asymptotic properties of statistical estimation of the Shannon entropy for distributions mixture
Statistical estimates of the Shannon entropy constructed by means of observations $X_1, \ldots, X_N$ having the same law as X are very important. They permit to estimate the mutual information and other related characteristics of a random vector X. Such estimates are widely used in machine learning, they are essential for tests concerning independence hypothesis for collections of random variables and they are employed in feature selection theory and various applications. The behavior of the Kozachenko - Leonenko estimates for the (differential) Shannon entropy, when the number of i.i.d. vector-valued observations tends to infinity, was studied by different authors. D.Pál et al. indicated the defects in the previous existing proofs of the asymptotic unbiasedness and L^2-consistency of these estimates. In a quite recent paper (Bulinski) we also turn to the mentioned results and establish them under wide conditions. To this end the analogues of the Hardy - Littlewood maximal function are proposed and employed. It is shown that our approach applies, in particular, to the entropy estimation of any nondegenerate Gaussian distribution. Moreover, we provide conditions to guarantee the validity of these new results for distributions mixture.

Anup Biswas (IISER, Pune)
Location of maximizers of eigenfunctions of nonlocal Schrodinger's equation
Eigenfunctions of the fractional Schrödinger operators in a domain D will be considered, and a relation between the supremum of the potential and the distance of a extrema of the eigenfunction from ∂D will be established. This results, in particular, extends a recent result of Rachh and Steinerberger (2017), to the fractional Schrödinger operators. We also generalize a celebrated Liebs theorem for fractional Schrödinger operators. As an application of these results we obtain a Faber-Krahn inequality for non-local Schrödinger operators. Extensions to more general non-local operators will also be discussed. This is based on a joint work with Jözsef Lőrinczi.

Sergey Berezin (Peter the Great St.Petersburg Polytechnic University)
On the RiemannHilbert problem, piecewise linear diffusions, and integral functionals
A connection between stochastic differential equations with piecewise linear coefficients and RiemannHilbert boundary value problems for analytic functions (RBVP) is going to be discussed. We present an approach that makes possible to derive probabilistic characteristics of some piecewise linear Ito diffusions as well as of some integral functionals of them. This approach has certain advantages over the standard one based on the FokkerPlanckKolmogorov or FeynmanKac equations.

In order to succeed, we investigate an evolution of the characteristic function of the diffusion using the so-called Pugachev equation, which is an analog of the Kolmogorov forward equation. In a piecewise linear case this equation comes down to the singular integral differential equation of the convolution type. The latter is called the PugachevSveshnikovequation and can be reduced to the RiemannHilbert linear conjugation problem for analytic functions. Some explicit results are going to be shown, and the approach used is going to be illustrated.

Sourav Sarkar (UC Berkeley)
Formation of large-scale random structure by competitive erosion
Begin with all sites of $\mathbb{Z}$ uncolored. A red particle performs simple random walk from 0 until it reaches a nonzero blue or uncolored site, and turns that site red; then, a blue particle performs simple random walk from 0 until it reaches a nonzero red or uncolored site, and turns that site blue. We prove that after$n$ red and n blue particles alternately perform such walks, the total number of colored sites is of order $n^{1/4}$. The resulting random color con figuration, after rescaling by $n^{1/4}$ and taking $n \to \infty$, has an explicit description in terms of alternating extrema of Brownian motion (the global minimum on a certain interval, the global maximum attained before that minimum, etc.). Joint work with Shirshendu Ganguly and Lionel Levine.

Vladislav Vysotsky (University of Sussex and PDMI)
Stationary overshoots of recurrent random walks
Take a one-dimensional recurrent random walk, and consider the sizes of its overshoots over the zero level. It turns out that this sequence, which forms a Markov chain, always has a unique invariant distribution of a simple explicit form. We will present a natural approach of finding this distribution and studying its properties using results of infinite ergodic theory. This approach also applies to overshoots of other Markov chains of similar type, the so-called oscillating random walks, whose increments are independent and sampled from one given distribution if the current position of the chain is positive and from the other distribution if otherwise. The initial interest to the problem is related to persistence of integrated random walks. The actual applications of our results include a limit theorem for the number of level-crossings of a general zero-mean random walk with finite variance. This is a joint work with Alex Mijatovic (Kings College London).

Fedor Sandomirskiy (National Research University Higher School of Economics (International Laboratory of Game Theory and Decision Making))
Fair division problems with desired and undesired items and their typical properties
Starting from the paper of Hugo Steinhaus (1948), where he considered a problem of fair division of a cake with different toppings among a group of friends having different tastes, economists became interested in fair division mechanisms. In practice we usually face a problem of distributing a family of indivisible goods (e.g., when a divorcing couple divides common assets) rather than cake-cutting problems. The setting is as follows. A finite set of goods G is to be allocated among a

finite set N of agents. An agent i has a value u_{ig} for a good $g \in G$. An allocation $z = (z_{ig})_{i \in N, g \in G}$ is a $N \times G$ bi-stochastic matrix, where element z_{ig} is the probability that agent i receives a good g; $U_i = U_i(z)$ is the expected value received by an agent. What allocations are considered as appropriate? There are two requirements usually imposed in the economic literature: Efficiency and Envy-freeness. An allocation z is called *Efficient* if there is no other allocation z' giving higher utilities to all agents. An allocation is *Envy-free* if $U_i(z) \geq U_i(\pi_{i,j}(z))$, where $\pi_{i,j}$ is a permutation of rows, corresponding to agents i and j. Efficient envy-free allocations always exist. Varian (1974) suggested the Competitive division rule having both properties and based on the ideas from economic General Equilibrium theory. A version of this rule is implemented on `spliddit.org`. In this talk we will discuss

- mathematical side of mentioned economic ideas
- typical properties of fair division mechanisms, when the matrix u is random and G is large
- **probabilistic, algorithmic and combinatorial** open problems corresponding to deterministic fair division mechanisms (i.e., $z_{ig} \in \{0, 1\}$)

Mathew Joseph (ISI, Bangalore)
An invariance principle for the stochastic heat equation
We will consider a weak approximation of the white noise driven stochastic heat equation, obtained by replacing the fractional Laplacian with the generator of a discrete time random walk, and approximating white noise by a collection of i.i.d. mean zero random variables. We will then describe how this approximation can be used to give a proof of the weak convergence of the scaled partition function of directed polymers in an intermediate disorder regime.

Mariia Platonova (Chebyshev Laboratory, St. Petersburg State University)
On the mean number of particles of branching random walk on $\mathbb{Z}^d$ with periodic sources of branching
We consider a continuous-time branching random walk on $\mathbb{Z}^d$ where the particles are born and die at a periodic set of points (the sources of branching). Spectral properties of the evolution operator of the mean number of particles are studied. In particular we prove that this operator has a positive spectrum which leads to an exponential asymptotic behavior of the mean number of particles when $t \to \infty$. Based on join work with K. Ryadovkin.

Deepan Basu (ISI, Kolkata)
Generalizations of Incipient Infinite Cluster in Planar Lattices and Slabs
We generalize Kesten's IIC in two ways. Firstly we establish multiple-arm IIC measure in planar lattices as certain local limits. Secondly, we prove the existence of IIC measure on slabs and then establish it as a certain limit. We also prove certain tools on slabs, namely quasi-multiplicativity

and RSW theorem, (which were previously known to be true for only the planar lattices) which function as key components of the proof. Joint work with Artem Sapozhnikov.

Rishideep Roy (IIM, Bangalore)
Branching random walk pressed against a hard wall
We wish to find bounds on the order of the probability of a branching random walk on a d-ary tree being positive at all vertices. We also wish to find the expected value at a typical vertex, under the condition that it is positive everywhere. The behaviour that we are considering is that of entropic repulsion for Gaussian fields which is its behaviour of drifting away when pressed against a hard wall. We show that the expected value is less than the expected maxima of BRW by a constant factor of $\log n$, which is a modification to the previous results on entropic repulsion of Gaussian fields(2D GFF in particular).

Alexander Bufetov (CNRS Steklov IITP NRUHSE)
Conditional measures of determinantal point processes: The Gibbs property and the completeness of reproducing kernels
Consider a Gaussian Analytic Function on the disk. In joint work with Yanqi Qiu and Alexander Shamov, we show that, almost surely, there does not exist a nonzero square-integrable holomorphic function with these zeros. By the Peres-Virag Theorem, zeros of a Gaussian Analytic Function on the disk are a determinantal point process governed by the Bergman kernel, and we prove, for general determinantal point processes, the conjecture of Russell Lyons and Yuval Peres that reproducing kernels sampled along a trajectory form a complete system in the ambient Hilbert space. The key step in our argument is that the determinantal property is preserved under conditioning. The problem, posed by Russell Lyons, of describing these conditional measures explicitly remains open even for the sine-process, and I will report on partial progress: the analogue of the Gibbs property for one-dimensional determinantal processes governed by integrable kernels. The talk is based on the preprint arXiv:1605.01400 as well as on the preprint arXiv:1612.06751 joint with Yanqi Qiu and Alexander Shamov.

Probal Chaudhuri (ISI, Kolkata)
Depth Regression
We consider a regression set up, where the response lies in a finite dimensional Euclidean space and the covariate is a random element in a metric space. In this set up, regression based on data depth yields information about the centre as well as other parts of the conditional distribution of the response given the covariate. We construct conditional central regions based on the depth function, which yield measures of conditional spread and skewness. A test for heteroscedasticity is developed based on the measure of conditional spread. The usefulness of the methodology is demonstrated in simulated and real datasets. This is joint work with Joydeep Chowdhuri.