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Direct Reconstruction of Complex Refractive Index Distribution from Boundary Measurements

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We obtain the reconstruction of the refractive index distribution of body based on the intensity and normal derivative of the intensity measurements. The Helmholtz equation is inverted either directly or indirectly through repeated implementation of the forward operator and its adjoint, for recovering the complex refractive index distribution. We do not adopt the procedure of recovery of phase (normally required for complete knowledge distribution). We derive certain sensitivity relations which is used for the easy computation of the Jacobian. Our procedure successfully reconstructs the real and imaginary parts of the complex refractive index from the measurement of the two data types derived from the complex amplitude at the boundary. Our other interest is the reconstruction of the spectroscopic variations of optical absorption coefficients and visco-elastic properties of a tissue which is extremely useful in diagnostic medicines. The research is on progress and some results are available.

1 Diffraction Tomography: Reconstruction based on boundary measurement of intensity and normal derivative of intensity

For the reconstruction of complex refractive index of objects at a spatial resolution comparable to wavelength of radiation used to interrogate the object, diffraction tomography is employed. Diffraction tomography provides inversion algorithms, based on Fourier diffraction theorem, such as filtered back propagation or direct Fourier method, under the assumption of weak scattering. In addition to these noniterative methods, there are also iterative methods employed for solving the inverse problem, which have provision to circumvent the weak scattering approximation. Under these schemes, the Helmholtz equation is inverted either directly or indirectly through repeated implementation of the forward operator and its adjoint, for recovering the complex refractive index distribution. In these two methods the data on the boundary is the complex amplitude which is the sum of the incident and the scattered field and a complete data set should have both the intensity and phase of the transmitted light. Whereas the intensity is easily detected by photo-detectors or charge coupled device (CCD) arrays, phase is not, which requires indirect and experimentally complex measurement methods. Since a complete recovery of the complex amplitude data is cumbersome, there have been many attempts to reconstruct the refractive index distribution directly from the measured intensity data. The forward operator under consideration reduces to

$$\nabla\cdot\nabla v(\vec{r})+2ik_0\theta\cdot\nabla v(\vec{r})-k_0^2 v(\vec{r})f(\vec{r})=k_0^2 f(\vec{r}) \tag{1}$$

with boundary conditions

$$v(\vec{r})=0 \quad on \quad L\cup L^- \qquad\qquad v(\vec{r})+\frac{\partial v(\vec{r})}{\partial n}=0 \quad on \quad L^+ \tag{2}$$

Here $f(\vec{r})$, is the perturbation in the refractive index to be reconstructed and $v(\vec{r})$, the multiplicative perturbation owing to scattering on the incident plane wave. We also need the adjoint equation

$$\nabla\cdot\nabla\psi(\vec{r})+2ik_0\theta\cdot\nabla\psi(\vec{r})-k_0^2\bar{f}(\vec{r})\psi(\vec{r})=0 \tag{3}$$

In the present work, we proposed an iterative reconstruction procedure which successfully gets back the real and imaginary parts of the complex refractive index from the measurement of the two data types derived from the complex amplitude at the boundary. The data types are *logarithm of intensity* ($ln(I)$) and the *normal derivative of the intensity* ($\frac{\partial I}{\partial n}$) which are intuitively selected keeping in mind the following facts: (i) Intensity of the transmitted light is affected primarily by the imaginary part of the object refractive index. (ii) The normal derivative of the intensity, which determines the intensity transport across the wavefront is controlled primarily by the curvature of the wavefront, which in turn is dependent on the real part of the refractive index of the object through which the light is propagated. The method which is iterative, repeatedly implements the forward propagation equation for light amplitude, the Helmholtz equation, and computes appropriate sensitivity matrices for these

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measurements. Appropriate sensitivity (reciprocity) relations are derived for an easy computation of the Jacobian. These are relations connecting the Greens' functions of forward and adjoint operators corresponding to the measurement data like:

$$\begin{aligned} &G^{\Gamma^{\delta}_{I},1}(m_0,r) = -\bar{G}^{\psi}_{R}(r,m_0),\ \bar{G}^{\Gamma^{\delta}_{I},2}(m_0,r) = -G^{\psi}_{R}(r,m_0) \\ &\bar{G}^{\psi}_{R}(r,m_0) = G^{\Gamma_{\frac{\partial I}{\partial n}},1}(m_0,r)\ G^{\psi}_{R}(r,m_0) = \bar{G}^{\Gamma_{\frac{\partial I}{\partial n}},2}(m_0,r). \end{aligned} \tag{4}$$

These relations helps us in the easy computation of the Jacobian matrices. The sensitivity matrices are computed solving the forward propagation equation for light and its adjoint. The results of numerical experiments show that the data types $ln(I)$ and $\frac{\partial I}{\partial n}$ reconstructed respectively the imaginary and real part of the object refractive index distribution. Moreover the imaginary part of the refractive index reconstructed from $\frac{\partial I}{\partial n}$ and the real part from $ln(I)$ failed to show forth the the object inhomogeneity. The value of the propagation constant, k, used in our simulations was fifty and this value resulted in smoothening of the inhomogeneities reconstructed. Thus we have shown that it is possible to reconstruct complex refractive index distribution directly from the measured intensity without having to first find the light amplitude, as is done in most of the currently available reconstruction algorithms of diffraction tomography.

2 Diffusing wave spectroscopy

Near infrared light has been increasingly used to probe soft tissue organs for early diagnosis of onset of diseases such as cancer. The property usually imaged is the spectroscopic variations of optical absorption coefficient in the range of 700-900 nm, which is done in diffuse optical tomography (DOT). The second property of interest for diagnostic imaging is the visco-elastic property of the tissue, which also shows forth an enormous increase due to the onset of disease such as cancer. For visco-elastic property measurement, usually movement of tissue particles is introduced by such means as remote palpation by focussed ultrasound beam, electromagnetic and piezo-electric vibrators, with which shear waves are sent through the body. The sinusoidal vibrations introduced by the ultrasound force causes the light interrogating the region to pick up a phase modulation . This phase modulation can be observed as a modulation on the intensity or the amplitude auto correlation($g_2(\tau)$ and $g_1(\tau)$ respectively) of the detected light on the boundary. The depth of modulation in $g_1(\tau)$ or $g_2(\tau)$ is related to the local visco-elastic properties of the region in the body insonified by the ultrasound radiation. In this work we try to understand the inverse pde problem associated with DCT and arrive at accurate method to reconstruct the coefficients of the pde. The first part of the work consists of establishing the continuity, coercivity, and differentiabilty of the bilinear form associated with the above pde. Thereafter we plan to establish accurate and easy method to evaluate the Jacobian(derivative after discretization) of the forward propagation equation for $g_1(\tau)$. Another major aim of the study is to arrive at data types which can be obtained from the measured $g_2(\tau)$ which can selectively discriminate moments of velocity distribution from mean squared displacement in the reconstruction. We also plan some experimental verification on tissue equivalent phantoms and also on small animals.

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