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Trajectory controllability of nonlinear integro-differential system

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Abstract

A stronger concept of complete (exact) controllability which we call *Trajectory Controllability* is introduced in this paper. We study the Trajectory Controllability of an abstract nonlinear integro-differential system in the finite and infinite dimensional space setting.

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1. Introduction

The concept of controllability (introduced by Kalman 1960) leads to some very important conclusions regarding the behavior of linear and nonlinear dynamical systems. Most of the practical systems are nonlinear in nature and hence the study of nonlinear

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systems is important. There are various notions of controllability such as complete controllability [10], approximate controllability [13], exact controllability [4–8,20], partial exact controllability [22], null controllability [21], local controllability [2], constrained controllability [15–18] and references cited in. A new notion of controllability, namely, Trajectory controllability (T-controllability) is introduced here for some abstract nonlinear integro-differential systems. In T-controllability problems, we look for a control which steers the system along a prescribed trajectory rather than a control steering a given initial state to a desired final state. Thus this is a stronger notion of controllability.

Under suitable conditions, the T-controllability of nonlinear system in finite dimensional case has been established in Section 2. Then the result is extended to infinite dimensional case in Section 3. We use the tools of monotone operator theory and set-valued analysis. We also use Lipschitzian and monotone nonlinearities with coercivity property in Section 3. Examples are provided to illustrate our results.

Remark 1.1. In practical applications, controls are always in some sense of constrained. Recently Klamka [16] studied the sufficient conditions for constrained local relative controllability of semilinear ordinary differential state equation in finite dimension with delayed controls using a generalised open mapping theorem where he assumed that the values of admissible controls are in a convex and closed cone with the vertex at zero. Also Klamka [15] proved the constrained exact controllability of first and second order systems in infinite dimension space. One can extend our system for second order and study *T*-controllability result.

2. T-controllability of finite-dimensional systems

Consider the nonlinear scalar system

$$\left.\begin{aligned} x'(t) &= a(t)x(t) + b(t,u(t)) + f\left(t, x(t), \textstyle\int_0^t g(t,s,x(s))\,ds\right),\\ x(0) &= x_0, \end{aligned}\right\} \tag{2.1}$$

for all $0 \le t \le T < \infty$. Here, $a(t)$ is an $\L^1$ function defined on $J=[0,T]$ and $b : J \times \mathbb{R} \mapsto \mathbb{R}$. For $t \in J$, the state $x(t)$ and the control $u(t)$ belong to $\mathbb{R}$. Further, $f : J \times \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}$ is a nonlinear function satisfying the Caratheadory conditions, i.e. f is measurable with respect to first argument and continuous with respect to second argument. Also, $g : \Delta \times \mathbb{R} \mapsto \mathbb{R}$ is a nonlinear function which also satisfies the Caratheadory conditions, where $\Delta = \{(t,s) \in J \times J; 0 \le s \le t \le T\}$.

Definition 2.1. The system (2.1) is said to be *completely controllable* on J if for any $x_0, x_1 \in \mathbb{R}$, and fixed T, there exists a control $u(\cdot) \in \L^2(J)$ such that the corresponding solution $x(\cdot)$ of (2.1) satisfies $x(T) = x_1$.

It may be noted that according to the above definition, there is no constraint imposed on the control or on the trajectory.

Remark 2.2. For the system (2.1), it is possible to steer any initial state x_0 to any desired final state x_1. But it does not give any idea about the path along which the system moves. Practically it may be desirable to steer the system from initial state x_0 to a final state x_1 along a prescribed trajectory. It may minimize certain cost involved in steering the system, depending upon the path chosen. It may also safe-guard the system. This motivates the study on the notion of T-controllability.

Let $\mathcal{T}$ be the set of all functions $z(\cdot)$ defined on $J=[0,T]$ such that $z(0)=x_0,\ z(t)=x_1$, $t\in J$ and z is differentiable almost everywhere.

Definition 2.3. The system (2.1) is said to be *T-controllable* if for any $z\in\mathcal{T}$, there exists a control $u\in L^2(J)$ such that the corresponding solution $x(\cdot)$ of (2.1) satisfies $x(t)=z(t)$ a.e.

Definition 2.4. The system (2.1) is *totally controllable* on J if for all subintervals $[t_i,t_f]$ of $[0,T]$, the system (2.1) is completely controllable.

Clearly, T-controllability $\Longrightarrow$ Total controllability $\Longrightarrow$ Complete controllability.

In the system (2.1), both control $u(\cdot)$ and state $x(\cdot)$ appear nonlinearly. First let us look at the following system where the control appears linearly:

$$\left.\begin{array}{l} x'(t)=a(t)x(t)+b(t)u(t)+f(t,x(t),\int_0^t g(t,s,x(s))\,ds),\\ x(0)=x_0, \end{array}\right\} \tag{2.2}$$

Assumptions [A1]. (i) The functions $a(t)$ and $b(t)$ are continuous on J.

(ii) $b(\cdot)$ do not vanish on J.

(iii) f is Lipschitz continuous with respect to second and third argument, i.e. there exist α_1,α_2 such that

$$|f(t,x_1,y_1)-f(t,x_2,y_2)|\le\alpha_1|x_1-x_2|+\alpha_2|y_1-y_2|$$

for all $x_1,x_2,y_1,y_2\in R,\ t\in J$.

(iv) g is L^1-Lipschitz continuous with respect to the third argument in the following sense:

$$\int_0^t |g(t,s,x(s))-g(t,s,y(s))|\,ds\le\beta|x(t)-y(t)|,\quad x,y\in\mathcal{T},(t,s)\in\Delta.$$

Under the above assumptions, one can easily construct the control explicitly to prove the T-controllability of the nonlinear system (2.2). To see this we proceed as follows:

For each control $u\in L^2(J)$, the existence and uniqueness of the solution for the system (2.2) follow from Assumptions [A1] by using the standard arguments.

Let $z(t)$ be a given trajectory in $\mathcal{T}$. We define a control function $u(t)$ by

$$u(t)=\frac{z'(t)-a(t)z(t)-f(t,z(t),\int_0^t g(t,s,z(s))\,ds)}{b(t)}.$$

With this control, (2.2) becomes,

$$x'(t)=a(t)x(t)+z'(t)-a(t)z(t)-f\left(t,z(t),\int_0^t g(t,s,z(s))\,ds\right)+f\left(t,x(t),\int_0^t g(t,s,x(s))\,ds\right),$$

$$x(0)=x_0.$$

Setting $w(t)=x(t)-z(t)$, we have

$$\left.\begin{aligned} w'(t) &= a(t)w(t)+f\left(t,x(t),\textstyle\int_0^t g(t,s,x(s))\,ds\right)-f\left(t,z(t),\textstyle\int_0^t g(t,s,z(s))\,ds\right),\\ w(0) &= 0. \end{aligned}\right\} \tag{2.3}$$

By using the transition function $\phi(t,s)=e^{\int_s^t a(s)\,ds}$ for the ordinary differential equation $y'(t)=a(t)y(t)$, (2.3) can be rewritten as

$$w(t)=\int_0^t \phi(t,s)\left[f\left(s,x(s),\int_0^s g(s,\tau,x(\tau))\,d\tau\right)-f\left(s,z(s),\int_0^s g(s,\tau,z(\tau))\,d\tau\right)\right]ds.$$

Thus

$$\begin{aligned} |w(t)| &\le \int_0^t |\phi(t,s)|\left[\alpha_1|x(s)-z(s)|+\alpha_2\left|\int_0^s g(s,\tau,x(\tau))\,d\tau-\int_0^s g(s,\tau,z(\tau))\,d\tau\right|\right]ds\\ &\le \int_0^t |\phi(t,s)|[\alpha_1|x(s)-z(s)|+\alpha_2\beta|x(s)-z(s)|]\,ds. \end{aligned}$$

That is,

$$|x(t)-z(t)|\le(\alpha_1+\alpha_2\beta)\int_0^t |\phi(t,s)\|x(s)-z(s)|\,ds.$$

Hence by Grownwall's inequality, it follows that

$$\|x(t)-z(t)\|=0.$$

This proves T-controllability of the system (2.2).

As remarked earlier in the above nonlinear system (2.2), the control $u(t)$ is appearing linearly. Let us now consider the case in which control as well as the state appear nonlinearly as in (2.1). We have following theorem.

Theorem 2.5. *Suppose that*

(i) $b(t,u)$ *is continuous.*
(ii) $b(t,u)$ *is coercive in the second variable, i.e.*

$$b(t,u)\to\pm\infty \text{ as } u\to\pm\infty,\quad t\in J.$$

(iii) *The function* f *is Lipschitz continuous in the second and third variable, uniformly in* t, *i.e. there exist* $\alpha_1>0$ *and* $\alpha_2>0$ *such that*

$$|f(t,x_1,y_1)-f(t,x_2,y_2)|\le\alpha_1|x_1-x_2|+\alpha_2|y_1-y_2|,\quad \forall x_1,x_2,y_1,y_2\in\mathbb{R},\ t\in J.$$

(iv) *The function* g *is Lipschitz in the third variable uniformly in* $(t,s)\in\Delta$, *i.e., there exists* $\beta>0$ *such that*

$$|g(t,s,x)-g(t,s,y)|\le\beta|x-y|\forall x,\quad y\in\mathbb{R},\ (t,s)\in\Delta.$$

Then the nonlinear system (2.1) *is T-controllable.*

Proof. For each fixed u, the existence and uniqueness of the solution of the system (2.1) follow from the Lipschitz continuity of the functions f and g. Moreover, this solution satisfies the integral equation

$$x(t) = \phi(t,0)x_0 + \int_0^t \phi(t,s)b(s,u(s))\,ds + \int_0^t \phi(t,s)f\left(s,x(s),\int_0^s g(s,\tau,x(\tau))\,d\tau\right)ds. \tag{2.4}$$

Let $z \in \mathcal{T}$ be the prescribed trajectory with $z(0)=x_0$. We want to find a control u satisfying

$$z(t) = \phi(t,0)x_0 + \int_{t_0}^t \phi(t,s)b(s,u(s))\,ds + \int_0^t \phi(t,s)f\left(s,z(s),\int_0^s g(s,\tau,z(\tau))\,d\tau\right)ds.$$

The above equation can be written as

$$z(t)-\phi(t,0)x_0-\int_0^t \phi(t,s)f\left(s,z(s),\int_0^s g(s,\tau,z(\tau))\,d\tau\right)ds = \int_0^t \phi(t,s)b(s,u(s))\,ds.$$

Differentiating with respect to t, we get

$$z'(t)-a(t)\phi(t,0)x_0-\int_0^t a(t)\phi(t,s)f\left(s,z(s),\int_0^s g(s,\tau,z(\tau))\,d\tau\right)ds-f\left(t,z(t),\int_0^t g(t,s,z(s))\,ds\right)$$
$$= \int_0^t a(t)\phi(t,s)b(s,u(s))\,ds + b(t,u(t)). \tag{2.5}$$

Eq. (2.5) can be written as

$$w(t) = \int_0^t k(t,s)w(s)\,ds + w_0(t), \tag{2.6}$$

where $w(t) = b(t,u(t))$, $k(t,s) = -a(t)\phi(t,s)$ and $w_0(t)$ is the left hand side of (2.5).

Eq. (2.6) is a linear Volterra integral equation of the second kind and it has a unique solution $w(t)$ for each given $w_0(t)$ (refer [19]). Hence it suffices to extract $u(t)$ from the solution $w(t)$. To extract $u(t)$, we use the technique of Deimling [11,12].

Consider the multi-valued function $G:[0,T]\to 2^{\mathbb{R}}$ defined by $G(t) = \{u \in \mathbb{R} : b(t,u) = w(t)\}$. Since $b(\cdot,\cdot)$ and $w(\cdot)$ are continuous, by hypothesis (ii) $G(t)$ is nonempty for all t and upper semi-continuous. That is, $t_n \to 0$ implies $G(t_n) \subset G(0)+\overline{B}_\varepsilon(0)$, $\forall n \geq n(\varepsilon,0)$. Further, G has compact values. Hence G is Lebesgue measurable and therefore has a measurable selection $u(\cdot)$. This function u is the required control which steers the nonlinear system along the prescribed trajectory $z(\cdot)$.

Hence proof is complete. □

Remark 2.6. (i) The control u obtained in Theorem 2.5 is measurable, may not be continuous. But, if we require control u to be continuous, we have to assume more stronger condition on $b(t,u)$.

(ii) If the nonlinear function $b(t,u)$ is invertible then $u(t)$ can be computed directly from $w(t)=b(t,u(t))$. For example, if $b(t,u)$ is strongly monotone, i.e. there exists $\beta>0$ such that

$$|b(t,u)-b(t,v)| \geq \beta|u-v|,$$

then there exists a unique u such that $b(t,u)=w$. Note that the strong monotonicity implies coercivity.

(iii) If $b(t,u)$ is coercive and monotonically increasing with respect to u, then it can be seen that $b(t,\mathbb{R})=\mathbb{R}$ and $b(t,u)=w(t)$ is solvable.

Example 2.7. Consider the nonlinear integro-differential system with the control term $b(t,u)=u|u|$.

$$\left.\begin{aligned}&x'(t)=a(t)x(t)+b(t,u(t))+\sin\big((x(t)+3\textstyle\int_0^t x(s)\,ds\big),\\&x(0)=x_0.\end{aligned}\right\}$$

The control term $b(t,u)$ is continuous and coercive. One can now verify f and g as in Theorem 2.5 to get T-controllability of the above system.

3. T-controllability of infinite-dimensional systems

In this section we consider a nonlinear integro-differential system defined in infinite dimensional space and generalize the results of Section 2. Let H and U be Hilbert spaces and consider following nonlinear integro-differential system:

$$\left.\begin{aligned}&w'(t)=Aw(t)+B(t,u(t))+F(t,w(t),\textstyle\int_0^t G(t,s,w(s)\,ds),\quad t\in J=[0,T],\\&w(0)=w_0,\end{aligned}\right\}\tag{3.1}$$

where the state $w(t)\in H$ and the control $u(t)\in U$, for each $t\in J$. The operator $A:\overline{\mathcal{D}(A)}\subset H\mapsto H$ is a linear operator not necessarily bounded. The maps $B:J\times U\mapsto H$, $G:\Delta\times H\mapsto H$ and $F:J\times H\times H\mapsto H$ are nonlinear operators, where $\Delta=\{(t,s)\in J\times J:0\le s\le t\le T\}$.

We make the following assumptions on (3.1).

Assumptions [I]. (i) Let A be an infinitesimal generator of a strongly continuous C_0-semigroup of bounded linear operators $S(t)$, $t\ge 0$. So there exist constants $M_1\ge 0$ and $w\in R^+$ such that

$$\|S(t)\|\le M_1e^{wt},\quad t\ge 0$$

and also let

$$\int_0^T\int_0^t\|S(t-s)\|^2\,ds\,dt<\infty.$$

(ii) B and G satisfy Caratheadory conditions, i.e. $B(t,\cdot):U\mapsto H$ is continuous for $t\in J$ and $B(\cdot,x):J\mapsto H$ is measurable for $x\in U$ and $G(t,s,\cdot):H\mapsto H$ is continuous $\forall(t,s)\in\Delta$ and $G(\cdot,\cdot,x):\Delta\mapsto H$ is measurable $\forall x\in H$.

(iii) F satisfies Caratheadory conditions like G.

(iv) B, G and F satisfy following growth conditions:

$$\|B(t,u)\|_H\le b_0(t)+b_1\|u\|_U\quad\forall u\in U,\quad t\in J,$$

$$\|G(t,s,x)\|\le q_0(t)+q_1\|x\|_H\quad\forall t\in J,\quad x\in H,$$

$$\|F(t,x,y)\|_H \le a_0(t) + a_1\|x\|_H + a_2\|y\|_H.$$

Under Assumptions [I], a mild solution of the system (3.1) satisfies the Volterra integral equation

$$w(t) = S(t)w_0 + \int_0^t S(t-s)B(s,u(s))\,ds + \int_0^t S(t-s)F\left(s,w(s),\int_0^s G(s,\tau,w(\tau))\,d\tau\right)ds. \tag{3.2}$$

Let $\mathcal{T}$ be the set of all functions $z \in L^2(J,H)$ which are differentiable and $z(0) = w_0$. We say that the system (3.1) is T-controllable if for any $z \in T$, there exists an L^2-function $u : J \mapsto H$ such that the corresponding solution w of (3.1) satisfies $w(\cdot) = z(\cdot)$ a.e.

We make the following additional assumptions on F and B.

Assumptions [II]. (i) $F(t,x,y)$ is Lipschitz continuous with respect to x and y, i. e. there exist constants $\alpha_1, \alpha_2 \ge 0$ such that

$$\|F(t,x_1,y_1) - F(t,x_2,y_2)\| \le \alpha_1\|x_1 - x_2\| + \alpha_2\|y_1 - y_2\|$$

for all $x_1, x_2, y_1, y_2 \in H,\ t \in J$.

(ii) $G(t,s,x)$ is Lipschitz continuous with respect to x, i.e. there exists a constant $\beta > 0$ such that

$$\|G(t,s,x) - G(t,s,y)\| \le \beta\|x - y\|, \quad x,y \in H,\ (t,s) \in \Delta.$$

(iii) B satisfies monotonicity and coercivity conditions, i.e.

$$\langle B(t,u) - B(t,v), u - v\rangle \ge 0, \quad \forall u,v \in U,\ t \in J$$

and

$$\lim_{\|u\| \to \infty} \frac{\langle B(t,u),u\rangle}{\|u\|} = \infty.$$

We now prove the T-controllability result for the system (3.1).

Theorem 3.1. *Under Assumptions* [I] *and* [II], *the nonlinear system* (3.1) *is T-controllable*.

Proof. Let z be any trajectory in $\mathcal{T}$. Following the proof of the Theorem 2.5, we look for a control u satisfying

$$z(t) - S(t)w_0 - \int_0^t S(t-s)F\left(s,z(s),\int_0^s G(s,\tau,z(\tau))\,d\tau\right)ds = \int_0^t S(t-s)B(s,u(s))\,ds.$$

Differentiating with respect to t, we get

$$\left[z^{'}(t) - AS(t)w_0 - \int_0^t AS(t-s)F\left(s,z(s),\int_0^s G(s,\tau,z(\tau))\,d\tau\right)ds - F\left(t,z(t),\int_0^t G(t,s,,z(s))\,ds\right)\right]$$

$$= \int_0^t AS(t-s)B(s,u(s))\,ds + B(t,u(t)). \tag{3.3}$$

Eq. (3.3) can be rewritten in the form

$$y(t) = \int_0^t k(t,s)y(s)\,ds + y_0(t), \tag{3.4}$$

where $y(t) = B(t,u(t))$, $k(t,s) = -AS(t-s)$ and $y_0(t)$ is the left hand side of (3.3).

Define an operator $K\colon L^2(J,H) \to L^2(J,H)$ by

$$(Ky)(t) = \int_0^t k(t,s)y(s)\,ds. \tag{3.5}$$

Assumption [I(i)] assures that K is a bounded linear operator [3]. Also, it can be easily proved that K^n is a contraction for sufficiently large n (refer [11,22]). Hence by generalized Banach contraction principle, there exists a unique solution y for (3.4) for given $y_0 \in L^2(J,H)$. Therefore, T-controllability follows if we can extract $u(t)$ from the relation:

$$B(t,u(t)) = y(t). \tag{3.6}$$

To see this, define an operator $N\colon L^2(I,H) \to L^2(I,H)$ by

$$(Nu)(t) = B(t,u(t)). \tag{3.7}$$

Assumptions [I(ii)–(iv)] imply that N is well-defined, continuous and bounded operator. Assumption [II(iii)] shows that N is monotone and coercive. A hemi-continuous monotone mapping is of type (M) (see [14, p. 78]). Therefore, by Theorem 3.6.9 of Joshi and Bose [14], the nonlinear map N is onto. Hence there exists a control u satisfying (3.6). The measurability of $u(t)$ follows as u is in $L^2(I,H)$. This proves T-controllability of the system (3.1). □

Corollary 3.2. *If F and G are Lipschitz continuous and B is strongly monotone, i.e. there exists* $\beta > 0$ *such that*

$$\langle B(t,u) - B(t,v), u-v \rangle \geq \beta \|u-v\|^2 \forall u, \quad v \in H, \ t \in J. \tag{3.8}$$

Then the system (3.1) *is T-controllable.*

Proof. The proof follows from the fact that the condition (3.8) implies Assumption [II(iii)]. □

Remark 3.3. We have not directly used the Assumptions [II(i)] and [II(ii)] of the Lipschitz continuity of f in the proof of Theorem 3.1. Actually, it is needed for the existence and uniqueness of the solution $w(\cdot)$ satisfying (3.2) for each control $u(\cdot)$. There are also other verifiable conditions for the uniqueness of the solution, in the literature (see [7]).

Example 3.4. Let Ω be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$. Consider the system

$$\left.\begin{aligned} &\frac{\partial y}{\partial t} = \Delta y + u(x,t) + \frac{1}{2}[\sin^2 x(t) + \sin y(t)] \quad \text{in } \Omega \times (0,T),\\ &y(x,0) = 0 \qquad\qquad \text{in} \quad \Omega,\\ &y(x,t) = 0 \quad \text{in } \partial\Omega \times (0,T). \end{aligned}\right\}$$

The above system can be put into the form of (3.1) by defining $Aw(t) = \Delta w(t)$ for all $w(t) \in \mathcal{D}(A)$, where $\mathcal{D}(A) = H^2(\Omega) \cap H_0^1(\Omega)$ is the domain of A and $H = U = L^2(\Omega)$. Here the control term $B(t,u(t)) = u(t)$ is linear. The above system is T-controllable under the assumptions on F and G as in the theorem.

In the one-dimensional case, say, $\Omega = (0,1)$, one can explicitly write $A : L^2(0,1) \to L^2(0,1)$ by $Aw = w''$, where $\mathcal{D}(A) = \{w \in H : w, w'$ are absolutely continuous, $w(0) = w(1) = 0\}$ and

$$Aw = \sum_{n=1}^{\infty} n^2 (w,w_n) w_n.$$

Here $w_n(s) = \sqrt{2}\sin ns$; $n = 1,2,3\ldots$ is the orthogonal set of eigenfunctions of A and (w,w_n) is the L^2 inner product. Further, A generates an analytic semigroup $S(t)$, $t \geq 0$ in H given by

$$S(t)w = \sum_{n=1}^{\infty} \exp(-n^2 t)(w,w_n)w_n, \quad w \in H.$$

Here $F(t,x(t),y(t)) = \frac{1}{2}[\sin^2 x(t) + \sin y(t)]$ and $G(t,s,y(s)) = \frac{1}{2}[\cos y(s)]$, both are Lipschitz continuous.

We now specialize Theorem 3.1 for the case $H = \mathbb{R}^n$. So we consider the following finite dimensional nonlinear system in $\mathbb{R}^n$:

$$\left.\begin{aligned} &w'(t) = A(t)w(t) + B(t, \quad u(t)) + F\left(t,w(t), \textstyle\int_0^t G(t,s,w(s))\,ds\right),\\ &w(0) = (w_0), \end{aligned}\right\} \tag{3.9}$$

where A, B, F and G are as in (3.1) with H replaced by $\mathbb{R}^n$. Therefore Theorem 3.1 can be specialized for the system (3.9) in $\mathbb{R}^n$. The following theorem can be proved as in Theorem 2.5.

Theorem 3.5. *Suppose that*

(i) *F is Lipschitz continuous with respect to x and y and G is Lipschitz continuous in x.*
(ii) *$B(t,u)$ satisfies*

$$\lim_{\|u\| \to \infty} \frac{\langle B(t,u),u \rangle}{\|u\|} = \infty.$$

Then the nonlinear system (3.9) *is T-controllable by a measurable control* $u : J \mapsto \mathbb{R}^n$.

Example 3.6. Consider the nonlinear two-dimensional system,

$$x_1'(t) = a_{11}x_1 + a_{12}x_2 + \sin\left(y_1(t) + 3\int_0^t y_1(s)\,ds\right) + \cos\left(y_2(t) + 3\int_0^t y_2(s)\,ds\right) + u_1^2,$$

$$x_1(0) = x_{01},$$

$$x_2'(t) = a_{21}x_1 + a_{22}x_2 + \cos\left(y_1(t) + 3\int_0^t y_1(s)\,ds\right) + \sin\left(y_2(t) + 3\int_0^t y_2(s)\,ds\right) + u_2^2,$$

$$x_2(0) = x_{02}.$$

It can be easily verified that the above system satisfies the hypotheses of Theorem 3.2, and hence it is T-controllable.

Concluding remarks: In this paper sufficient conditions for T-controllability of semilinear integro differential system in finite and infinite dimension spaces are proved by using measurable selections, generalised Banach contraction principle and monotone operatory theory.

The method presented here is quite general and covers wide class of semilinear dynamical control systems. Similar results may be proved for second order systems and semilinear dynamical control inclusions with delay arguments.

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