Math 232 Homework 1

Due on: August 16th 2017

1. Let P be a polygon in $\mathbb{R}^2$ with an even number of sides. Suppose that the sides are identified in pairs in some pattern.
   (a) Prove that the quotient space is a closed surface.
   (b) Give two *different* polygons or patterns of identifications that result in a surface of genus 2.

2. Show that for a topological space X, the following conditions are equivalent:
   (a) Every continuous map $S^1 \to X$ is homotopic to a constant map, with image a point.
   (b) Every continuous map $S^1 \to X$ extends to a continuous map $D^2 \to X$.
   (c) $\pi_1(X, x_0) = \{1\}$ for all $x_0 \in X$.

3. (a) Given a space X and a path-connected subspace A containing the basepoint x_0, show that the map $i_* : \pi_1(A, x_0) \to \pi_1(X, x_0)$ induced by the inclusion map $i : A \to X$ is surjective if and only if every path in X with endpoints in A is homotopic to a path in A.
   (b) What is an example of (X, A, x_0) above which does *not* satisfy the above property of the homomorphism i_* being surjective?