PROBLEM SET 3 FOR UM 102/UMA102

MAINLY ON DETERMINANTS.

(1) Let $\det \begin{pmatrix} x & y & z \\ 3 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} = 1$, compute the determinant of each of the following matrices:

$$\begin{pmatrix} 2x & 2y & 2z \\ \frac{3}{2} & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \begin{pmatrix} x & y & z \\ 3x+3 & 3y & 3z+2 \\ x+1 & y+1 & z+1 \end{pmatrix}, \begin{pmatrix} x-1 & y-1 & z-1 \\ 4 & 1 & 3 \\ 11 & 1 & 1 \end{pmatrix}.$$

(2) Prove that $\det \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} = (a-b)(b-c(c-a)$ and find formulas for $\det \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{pmatrix}$ and $\det \begin{pmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{pmatrix}$.

(3) Is $\det(A+B) = \det A + \det B$ in general?

(4) Let A be an $n \times n$ matrix. Consider the $2n \times 2n$ matrix $\begin{pmatrix} A & 0 \\ 0 & I \end{pmatrix}$. Show that its determinant is $\det A$.

(5) Let A be an $n \times n$ matrix and B be an $m \times m$ matrix. Consider the $(m+n) \times (m+n)$ matrix $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$. Show that its determinant is $\det A \det B$.

(6) Professor Bhattacharyya writes his office and home phone numbers as a 8×1 matrix A and a 1×8 matrix B respectively. Help him compute $\det(AB)$.

(7) An $n \times n$ real matrix A is called anti-symmetric if $A = -A^t$ where A^t is the matrix whose (i,j)th entry is the (j,i)th entry of A. Show that an anti-symmetric matrix has determinant 0 if n is odd.

(8) Prove that the determinant of

(a) the $2n \times 2n$ matrix $\begin{pmatrix} I & X \\ 0 & I \end{pmatrix}$ is 1 where X is an $n \times n$ matrix.

(b) the $2n \times 2n$ matrix $\begin{pmatrix} A & B \\ 0 & D \end{pmatrix}$ is $\det A. \det D$.

(c) the $2n \times 2n$ matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is $\det A. \det(D - CA^{-1}B)$ if A is invertible. Find a similar formula if D is invertible instead of A.