Problem Set

1. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $f(r, \theta, z) = (x, y, z)$

where $x = r\cos\theta$, $y = r\sin\theta$, $z = z$. Let

$g(x, y, z) = (x^2+y^2,\ x/y,\ 3z)$. Find the Jacobian matrix of $g(f(r, \theta, z))$.

2. The equations $u+v = x^2+y^2$ and $u-v = x^2-2xy^2$ define x and y implicitly as functions of u and v.

(a) Find $\frac{\partial x}{\partial u}$ and $\frac{\partial y}{\partial u}$ at $(u, v) = (-1, 6)$.

(b) If $z = \ln(y^2-x^2)$, find $\frac{\partial z}{\partial u}$ at $(u, v) = (-1, 6)$.

3. Find minima, maxima and saddle points for the following functions.

$a(x,y) = 3x^3 + 3x^2y - y^3 - 15x$

$b(x,y) = e^{x-y}(x^2 - 2y^2)$

$c(x,y) = e^{2x}(x + y^2 + 2y)$

$d(x,y) = x^3 - 4xy + 2y^2$

$e(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.

$f(x,y) = x^3 + y^3 - 3xy$

$g(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

$h(x,y) = x^3 + 8y^3 - 6xy + 5$

$i(x,y) = xy \ln(x^2+y^2)$,

$j(x,y) = x^3 + 3xy^2 - 15x - 12y$.

4. Find the largest values, the smallest values and the saddle points of the following functions in the given regions.

a $(x,y) = x^2y(4-x-y)$ in a triangle with sides along lines $x=0$, $y=0$, $x+y+6=0$,

b $(x,y) = x^2+y^2-xy+x+y$ in a triangle with sides along lines $x=0$, $y=0$, $x+y+3=0$,

c $(x,y) = x^2+2xy-4x+8y$ in a rectangle with sides along the lines $x=0$, $x=1$, $y=0$, $y=2$.

d $(x,y) = x^3+y^3-9xy+27$ in a square with sides along lines $x=0$, $x=1$, $y=0$, $y=1$,

$e(x,y) = xy$ in a square given by equation $|x|+|y|=1$,

$f(x,y) = xy$ in the closed unit circle $x^2+y^2 \leq 1$,

$g(x,y) = 2x^2 - 2y^2$ in the circle $x^2+y^2 \leq 4$,

$h(x,y) = x+y$ " " " .

$i(x,y) = x^2 - 2xy + 2y^2 - 2y$ in a region bounded by $y = \frac{1}{2}x^2$ and $y=2$.