Exercise Set VII

1. Let $F(x,y) = x^2 y$ and $G(x,y) = (x+y)^2$. The anticlockwise path along the perimeter of the triangle whose vertices are located at $(0,0)$, $(1,0)$ and $(0,1)$ is denoted by C. Evaluate the integral

$$\int_C F\,dx + G\,dy.$$

2. Let $F(x,y) = x^2 + xy$ and $G(x,y) = (x+y)^3$. The anticlockwise path along the perimeter of the square whose vertices are located at the points $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ is denoted by C. Evaluate the integral $\int_C F\,dx + G\,dy$.

3. Let $F(x,y) = (2xy^2 + \cos x,\ 2x^2 y - \sin y)$. Find an $\mathbb{R}$-valued function $\varphi(x,y)$ such that $\nabla \varphi = F$. Using this or otherwise find $\int_C F\,dr$ where C is the arc of the circle with equation $x^2+y^2 = \frac{\pi^2}{4}$, $y \geq 0$ from $(-\frac{\pi}{2}, 0)$ to $(\frac{\pi}{2}, 0)$.

4. Evaluate the integral $\int_{(-1,1)}^{(3,3)} (y+x)\,dx + (y-x)\,dy$ along the curve ~~with the~~

$$x = 2t^2 - 3t + 1, \quad y = t^2 - 1.$$

5. Evaluate the line ~~of~~ integral $\displaystyle\int_{(5,0)}^{(0,5)} (2x+y)\,ds$

where s is the arclength along the quarter circle with equation $x^2+y^2=25$.

6. Consider the ellipse $(x-1)^2 = 16(1-y^2)$. Let the ellipse meet the positive y and x axes at A and B respectively. Let C be the clockwise curve from A to B. Evaluate $\displaystyle\int_C (x^2+xy)\,dx + \left(y^2+\tfrac{1}{2}x^2\right)dy$ and

$$\int_C \left(y^3\,dx + \frac{1}{16}(x-1)^3\,dy\right).$$

7. Let C denote the boundary of

$$R = \{(x,y) : x+y \geq 0 \text{ and } x-y \leq 0 \text{ and } x^2+y^2 \leq 2\}.$$

Evaluate $\displaystyle\int_C xy\,dx + x^2\,dy$ when C is traced anticlockwise.

8. Questions 1 to 11 of Exercise 10.5 of Apostol, Calculus vol. II, page 328.

9. Questions 1 to 12 of Exercise 10.18 of page 345 of Calculus vol. II, Apostol.