HW 3

1. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ and $g : \mathbb{R}^3 \to \mathbb{R}^2$ be given by the equations $f(x) = (e^{2x_1+x_2}, 3x_2 - \cos(x_1), x_1^2 + x_2 + 2)$ and $g(y) = (3y_1 + 2y_2 + y_3^2, y_1^2 - y_3 + 1)$. If $F(x) = g \circ f(x)$, find $DF_{x=0}$ and if $G(y) = f \circ g(y)$, find $DG_{y=0}$.

2. Let $F(x, y) = f(x, y, g(x, y))$ where $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ are differentiable. Find DF in terms of the partials of f and g. If $F(x, y) = 0 \ \forall \ x, y$, find g_x, g_y in terms of the partials of f.

3. Give a counterexample to the mean value theorem for vector-valued functions.

4. Suppose the matrix-valued function $[F] : \mathbb{R}^4 \to Mat_{4\times 4} = \mathbb{R}^{16}$ is smooth and its image lies in the set of skew-symmetric matrices. Prove that in general, there is no smooth $g : \mathbb{R}^4 \to \mathbb{R}^4$ such that $[F]_{ij} = \frac{\partial g_i}{\partial x_j} - \frac{\partial g_j}{\partial x_i} \ \forall \ i, j$.