HW 5

1. Let $F(x, y, z) : \mathbb{R}^3 \to \mathbb{R}^2$ be defined as $F(x, y, z) = (x^2 + y^2 + z^2, xyz)$. Prove that there exists a neighbourhood of $(3, 1)$ such that for every (α, β) in this neighbourhood, there exists (a, b, c) such that $F(a, b, c) = (\alpha, \beta)$.

2. Let $f : U \subset \mathbb{R}^n \to \mathbb{R}$ (where U is open) be a C^1 function. Suppose $\nabla f(a) \neq 0$ and $f(a) = 0$. Prove that there exists a neighbourhood $V \subset U$ of a such that the tangent plane at a does not intersect the portion of the level set $f(x) = 0$ lying in this neighbourhood.