HW 9

1. Solve $x'' = e^x$ explicitly.
2. $x'' - x + x^3 = 0$: Here $E = \frac{1}{2}(x')^2 - \frac{x^2}{2} + \frac{x^4}{4}$. Prove that
   (a) For $-\frac{1}{4} < E < 0$: The solutions are periodic surrounding each of the two equilibrium points of length $a + b$ where $V(1 + b) = E = V(1 - a)$.
   (b) For $E = 0$: Solve the equation explicitly and conclude that there are two orbits. Deduce their behaviour as $t \to \pm\infty$.
   (c) For $E > 0$: We still get periodic orbits but restricted to $[-b, b]$ where $V(\pm b) = E$ and $b > \sqrt{2}$.
3. For $2E = 4k = (x')^2 + 2k(1 - \cos(x))$ with regard to the pendulum equation, we see that $2k(1 + \cos(x)) = (x')^2$. Solve this equation explicitly and deduce the behaviour of the orbits.