HW 3 (40 points) - To be handed by Thursday, Sept 5 in the class or by email

1. (20 points) Prove that $V^k = \gamma^k_{,s}$ can be extended locally to a smooth vector field, if γ is an admissible variation. Also prove that the variation vector field is continuous in s, t.

2. (10 points) Find the transformation rule for Christoffel symbols under change of coordinates. Also prove that the geodesic equation is invariant under changes of coordinates.

3. (10 points) Let $S \subset \mathbb{R}^3$ be a compact submanifold such that there exists a smooth immersion $\vec{r} : U \subset \mathbb{R}^2 \to \mathbb{R}^3$ that takes U homeomorphically to a relatively open subset of S. Suppose the image misses a set of measure zero in S. Let vol be the volume form of the induced Riemannian metric on S (from the Euclidean metric on $\mathbb{R}^3$). Prove that $\int_S vol_g = \int_U \|\vec{r}_u \times \vec{r}_v\| dudv$.