HW 7 (40 points) - To be handed by Friday, Oct 18 in the class or by email

1. (10 points) Prove that if p is a point on (M, g) then there is a $1 >> \delta > 0$ such that on the geodesic ball $U_{p,\delta}$ (a diffeomorphic image of a ball in the tangent space under the exponential map), if $\gamma(t) : [0, 1] \to M$ is a geodesic connecting any two $x, y \in U_{p,\delta}$, then $\max_{t\in[0,1]} r \circ \gamma(t) = \max(r(x), r(y))$.

2. (30 points) Let (M, g) be a compact Riemannian manifold. Prove that every non-trivial free homotopy class in M is represented by a closed geodesic that has minimum length among all admissible loops in the given free homotopy class. (Problem 6-17 in Lee.)