Homework 4 (due 23/Sep/2013)

Try all the exercises. Submit only those marked with an asterisk (*).

1. (*) Place r distinguishable balls in m distinguishable bins at random. Count the number of balls in the first bin.

1. Write the probability space and the random variable described above.
2. Find the probability mass function of the number of balls in the first bin.
3. Find the expected value of the number of balls in the first bin.

2. Let X be a random variable with distribution (CDF) F and density f.

1. Find the distribution and density of the random variable $2X$.
2. Find the distribution and density of the random variable $X+5$.
3. Find the distribution and density of the random variable $-X$.
4. Find the distribution and density of the random variable $1/X$.

3. For a pmf $f(\cdot)$, the *mode* is defined as any point at which f attains its maximal value (i.e., t is a mode if $f(t) \geq f(s)$ for any s). For each of the following distributions, find the mode(s) of the distribution and the value of the pmf at the modes.

1. $\text{Bin}(n, p)$.
2. $\text{Pois}(\lambda)$.
3. $\text{Geo}(p)$.

4. For a pdf $f(\cdot)$, the *mode* is defined as any point at which f attains its maximal value (i.e., t is a mode if $f(t) \geq f(s)$ for any s). For each of the following distributions, find the mode(s) of the distribution and the value of the pmf at the modes.

1. $N(\mu, \sigma^2)$.
2. $\text{Exp}(\lambda)$.
3. $\text{Gamma}(\nu, 1)$.

5. (*) Let F be a CDF. For each $0 < q < 1$, the *q-quantile(s)* of F is any number $t \in \mathbb{R}$ such that $F(s) \leq q$ if $s < t$ and $F(s) \geq q$ if $s > t$.

1. If F is the CDF of $\text{Exp}(\lambda)$ distribution, find its q-quantile(s).
2. If F is the $N(0,1)$ distribution, use the normal tables to find the unique q-quantile for the following values of q: $0.01, 0.1, 0.25, 0.5, 0.75, 0.9, 0.99$.
3. If F is the $\text{Geo}(0.02)$ distribution, find a q-quantile for $q = 0.01, 0.25, 0.5, 0.75, 0.99$.