PROBABILITY AND STATISTICS - HOMEWORK 1

DUE DATE: 17/AUG/2016 (IN CLASS)

In each of the following problems, first write the probability space (i.e., the sample space and elementary probabilities) and only then proceed to calculate probabilities. Homework problems are marked blue (but trying all problems is strongly recommended).

1. In a party there are N unrelated people. Their birthdays are noted (ignore leap years and assume that a year has 365 days). Find the probability of the event that no two of them have the same birthday. Get the numerical value for $N = 20$ and $N = 30$.

2. A deck of 52 cards is shuffled well and 3 cards are dealt. Find the probability of the event that all three cards are from distinct suits.

3. Place r_1 indistinguishable blue balls and r_2 indistinguishable red balls into m bins uniformly at random. Find the probability of the event that the first bin contains balls of both colors.

4. A coin with probability p of turning up H (assume $0 < p < 1$) is tossed till we get a TH or a HT (i.e., two consecutive tosses must be different, eg., TTH or $HHHT$). Find the probability of the event that at least 5 tosses are required.

5. A drunken man return home and tries to open the lock of his house from a bunch of n keys by trying them at random till the door opens. Consider two cases: (1) He is so drunk that he may try the same key several times. (2) He is moderately drunk and remembers which keys he has already tried.In both cases, find the probability of the event that he needs n or more attempts to open the door.

6. Let $\mathbf{x} = (0, 1, 1, 1, 0, 1)$ and $\mathbf{y} = (1, 1, 0, 1, 0, 1)$. A new 6-tuple $\mathbf{z}$ is created at random by choosing each z_i to be x_i or y_i with equal chance, for $1 \leq i \leq 6$ (A toy model for how two DNA sequences can recombine to give a new one). Find the probability of the event that $\mathbf{z}$ is identical to $\mathbf{x}$.

7. From a group of W women and M men, a team of L people is chosen at random (of course $L \leq W + M$). Find the probability of the event that the teams consists of exactly k women.

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8. Place r distinguishable balls in m labelled bins in such a way that each bin contains at most one ball. All *distinguishable* arrangements are deemed equally likely (this is known as Fermi-Dirac statistics). Find the probability that the first bin is empty.

9. A box contains $2N$ coupons labelled $1, 2, \ldots, 2N$. Draw k coupons (assume $k \leq N$) from the box one after another (1) with replacement, (2) without replacement. Find the probability of the event that no even numbered coupon is in the sample.

10. In a class with 108 people, one student gets a joke by e-mail. He/she forwards it to one randomly chosen classmate. The recipient does the same - chooses a classmate at random (could be the sender too) and forwards it to him/her. The process goes on like this for 20 steps and stops. What is the probability that the first person to get the mail does not get it again? What is the chance that no one gets the e-mail more than once?