PROBLEM SET 4

DUE ON SEPTEMBER 25 IN TUTORIALS

Submit only those coloured blue. Those in brown may be safely omitted and are meant for the mathematically inclined. Those in black are for practise. Those in white will appear in exams.

Problem 1. (1) Let $f(k) = \frac{1}{k(k+1)}$ for integer $k \geq 1$. Show that f is a pmf and find the corresponding CDF. What are its mean and median?

(2) Let $\alpha > 0$ and set $F(x) = 1 - \frac{1}{x^\alpha}$ for $x \geq 1$ and $F(x) = 0$ for $x < 1$. Show that F is a CDF and find the corresponding density function (this is known as the *Pareto* distribution). What are its mean and median?

Problem 2. A coin (not necessarily fair) is tossed till the kth head shows up. Let X be the number of tosses required. Find the pmf of X. [*Hint:* If $k = 1$, then $X \sim \text{Geo}(p)$, as discussed in class.]

Problem 3. The most likely value of a random variable is called its mode. More precisely, any value of t that maximizes $f(t)$, where f is the pmf or pdf of X (if they exist) is called a mode.

(1) Find the mode of $\text{Pois}(\lambda)$ distribution.

(2) Find the mode of the $\text{Gamma}(\alpha, 1)$ density.

Problem 4. You are assigned the task of making two six-sided dice (not necessarily uniform) with numbers of your choice on them. Both dice are thrown and the total of the two numbers that turn is recorded as X. The objective is that X should be equally likely to be 2 or 3 or $\ldots$ 11 or 12. Is it possible? [*Remark:* To understand the problem, show that the task is possible if the dice are three sided and the goal is that the sum should be equally likely one of 2,3,$\ldots$,10.]

Problem 5. A (hypothetical) mathematicain who needed random numbers pulled open a table of Fibonacci numbers and decided to use the first digits (which may take values $1, 2, \ldots, 9$) as his random numbers. Perform this experiment with the first 100 Fibonacci numbers (given by 1 ,1, 2, 3, 5, 8, 13,$\ldots$, where each number is the sum of the previous two) and see if you get uniform random numbers between 1 and 9. Check how the actual frequencies match again the pmf $f(k) = \log_{10}(k+1) - \log_{10}(k)$, for $k = 1, 2, \ldots, 9$.