PROBLEM SET 5

NOT FOR SUBMISSION

Submit only those coloured blue. Those in brown may be safely omitted and are meant for the mathematically inclined. Those in black are for practise. Those in white will appear in exams.

Problem 1. Given a random number generator that outputs random numbers from Exp(1) distribution, describe how you would generate random numbers from the following distributions. explicit description of how you would simulate random variables from the following distributions.

(1) Geometric distribution: Geo(p).

(2) Cauchy distribution with density $f(x) = \frac{1}{\pi(1+x^2)}$ for $x \in \mathbb{R}$.

(3) Beta$(1/2, 1/2)$ distribution with density $\frac{1}{\pi\sqrt{x(1-x)}}$.

Problem 2. In each of the following situations, the distribution of the random variable X is given. Find the distribution of Y (it is enough to find the density of Y).

(1) $X \sim$ Unif$[0,1]$ and $Y = \sin^{-1}(X)$.

(2) $X \sim$ Unif$[0,1]$ and $Y = \cos^{-1}(X)$.

[**Note:** We define $\sin^{-1}$ to take values in $[-\pi/2, \pi/2]$ and $\cos^{-1}$ to take values in $[0, \pi]$. In the third part, observe that $f(x) = x^2$ is not a one-one function, so the formula given in the notes does not apply directly].

Problem 3. Let $U \sim$ Uniform$[0,1]$. Find the density and distribution functions of (a) U^p (where $p > 0$), (b) $U/(1-U)$, (c) $\log(1/U)$, (d) $\frac{2}{\pi}\arcsin(U)$.

Problem 4. Let $Z \sim N(0,1)$. Show that $\mathbf{P}\{a-1 < Z < a+1\}$ is maximized when $a = 0$. What if $Z \sim$ Exp(1)?

Problem 5. If f is the density of X, write down the density of the following random variables. (a) $aX + b$ (where $a > 0$ and $b \in \mathbb{R}$), (b) X^3, (c) $1/X$.