MID-TERM EXAMINATION (UM 201)

03/OCTOBER/2018, 2:00-4:30 PM

Instructions to students

(1) Students are required to sit in their assigned seats only.
(2) Mobile phones/any form of communication devices are strictly prohibited in the exam rooms. It is best to leave the mobile phone in the hostel itself before coming for the exam.
(3) Mere possession of a mobile phone/communication device inside the examination hall or during the exam will be treated as a case of cheating (absolutely no excuses). In such cases, suitable action will be taken.
(4) No form of unfair means (including talking to another student, copying from another student's paper, copying from any books, notes, cheat sheets, etc., use of mobile phone/communication devices) during the examination will be tolerated. A student found resorting to any form of unfair means during the examination, will be given F grade in that course as a minimum punishment. No appeals will be accepted.
(5) Abetting a student to resort to any form of unfair means will also be considered as an unfair practice. In this instance as well, the student abetting another student will be given F grade in that course. No appeals will be accepted. In case the student who is abetting another student is from a different class/batch, suitable action will be taken on such a student.
(6) Before answering the questions, the student must write his/her name and student registration number on the answer script.
(7) When an additional sheet is taken by the student, the student must write his/her name and serial number, sign the additional sheet and must get it countersigned by the invigilator.
(8) **Calculators are strictly prohibited.**
(9) If a student is found with a mobile phone/communication device while taking a break to use the washroom when the exam is in progress, it will be treated as a case of cheating. Irrespective of whether the student is using the mobile phone/communication device or not, the same penalty as in item 4 will be applicable.

Exam-specific instructions

- **Start the answer to each of the five main questions on a new page.**
- **Answer as many questions as you wish - the maximum you can score is 50.**
- **Justify your answers but write succinctly. Marks will be cut if you write nonsense in addition to the correct answer.**
- **Ask for clarification in a question only if there is ambiguity in the wording - not to confirm if your answers are correct or to ask for definitions of terms already defined in class.**

Problem 1. (**12 marks**) For each statement, state whether it is true or false and justify.

(1) If $A_1, \ldots, A_5$ are events such that $\frac{1}{20} \le \mathbf{P}(A_i) \le \frac{1}{10}$ for all i, then $\frac{1}{20} \le \mathbf{P}\{A_1 \cup \ldots \cup A_5\} \le \frac{1}{2}$.

(2) If B_1, B_2 are disjoint events having positive probability, then $\mathbf{P}(A \mid B_1) + \mathbf{P}(A \mid B_2) = \mathbf{P}(A \mid B_1 \cup B_2)$ for any event A.

(3) Suppose A, B, C are (mutually) independent events and $D = A \cup B$. Then C and D are independent events.

(4) If $X \sim \text{Bin}(10, \frac{1}{2})$ and $Y \sim \text{Bin}(11, \frac{1}{2})$, then $\mathbf{P}\{X \ge 5\} \le \mathbf{P}\{Y \ge 5\}$.

Problem 2. (**12 marks**) Give the final answer to the following questions and justify.

(1) What is the median of the *Rayleigh density* $xe^{-\frac{1}{2}x^2}$ (for $x > 0$)?

(2) If 5 distinguishable balls are thrown into 4 labelled bins uniformly at random, what is the probability that there are exactly two empty bins?

(3) if A, B, C are events with probabilities 0.9, 0.8, 0.7, respectively, what is the minimum possible value of the probability of $A \cap B \cap C$?

(4) A fair die is thrown and if the number k turns up, a fair coin is tossed k times. If the total number of heads is 4, what is the chance that the die turned up 6 in the first place?

Problem 3. (**10 marks**) Balls are thrown one after another (uniformly at random) into two bins. The experiment stops when there is no empty bin. Let X be the total number of balls thrown.

(1) Find the pmf and CDF of X.

(2) Find the mean (expectation) and all the medians of X.

Problem 4. (**10 marks**) A random number generator gives random numbers according to Exp(1) distribution. Explain as explicitly as possible how you would use it to generate random numbers from the following distributions.

(1) Geo$(1/2)$ distribution.

(2) The *logistic distribution* having density $\frac{e^x}{(1+e^x)^2}$ for all $x \in \mathbb{R}$.

Problem 5. (**10 marks**) For $\lambda > 0$, let X_λ be a Pois(λ) random variable. For $n \ge 1$, let Y_n be a Gamma$(n, 1)$ random variable. Show that $\mathbf{P}\{X_\lambda \ge n\} = \mathbf{P}\{Y_n \le \lambda\}$.