HW 6 (to be submitted by Apr 8)

1. (9 marks) Show that covectors/1-forms $\omega_1, \ldots, \omega_k$ on a finite-dimensional vector space are linearly dependent iff $\omega^1 \wedge \ldots \omega^k = 0$.

2. (6 marks) Compute $F^*\omega$ in the following examples. ($F : M \to N$ is a given smooth map and ω is a given smooth form-field on N)

   (a) $M = N = \mathbb{R}^2$, $F(s,t) = (st, e^t)$, $\omega = xdy - ydx$.

   (b) $M = N = \mathbb{R}^3$, $F(u,v,w) = (u^2, uv, uvw)$, $\omega = xdy \wedge dz + y^2 dz \wedge dx$.

3. (10 marks) Prove that there is a smooth vector field on $\mathbb{R}^n$ whose associated time $t = 1$ diffeomorphism from $\mathbb{R}^n$ to itself interchanges a given pair of points (p, q).